APPLICABILITY OF FLUIDIC CONTROLS TO A
RANKINE-CYCLE AUTOMOTIVE ENGINE

FINAL REPORT

FOR

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Research Grant No. NGR 05-017-023

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The University of Santa Clara • California

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By

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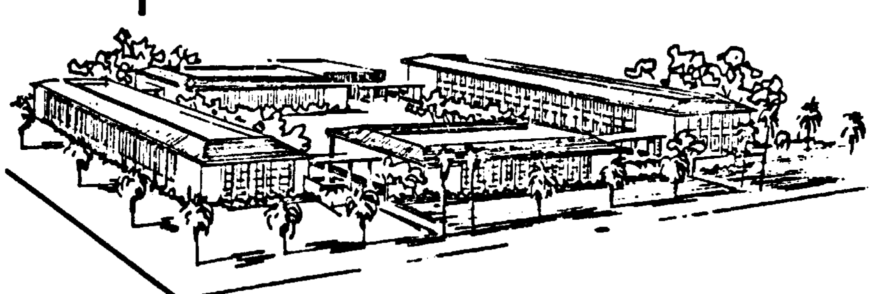

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ABSTRACT

Rankine-cycle performance with three different fluids is evaluated: water, CP-34, and Freon TF. Performance with CP-34 and a reciprocating expander using fluidic flow diverters instead of mechanical valves is examined. The control criteria for the boiler and feed pump are also investigated.

It is concluded that the use of fluidic flow diverters in place of mechanical valves is not feasible and that any further effort to apply fluidic controls to automotive engines should be in information sensing and processing rather than in direct control of power.

CONTENTS

I. INTRODUCTION

With the recognition that the automobile is a major contributor to air pollution, interest has been generated in a low pollution automotive power plant. Continuous combustion processes are capable of achieving lower emission levels than intermittent processes, for in continuous combustion processes the reaction is not quenched. Consequently, there is renewed interest in an automotive power plant based on the Rankine cycle.

The basic elements of a Rankine cycle are shown in Fig. 1. Heat is generated in a continuous combustion process

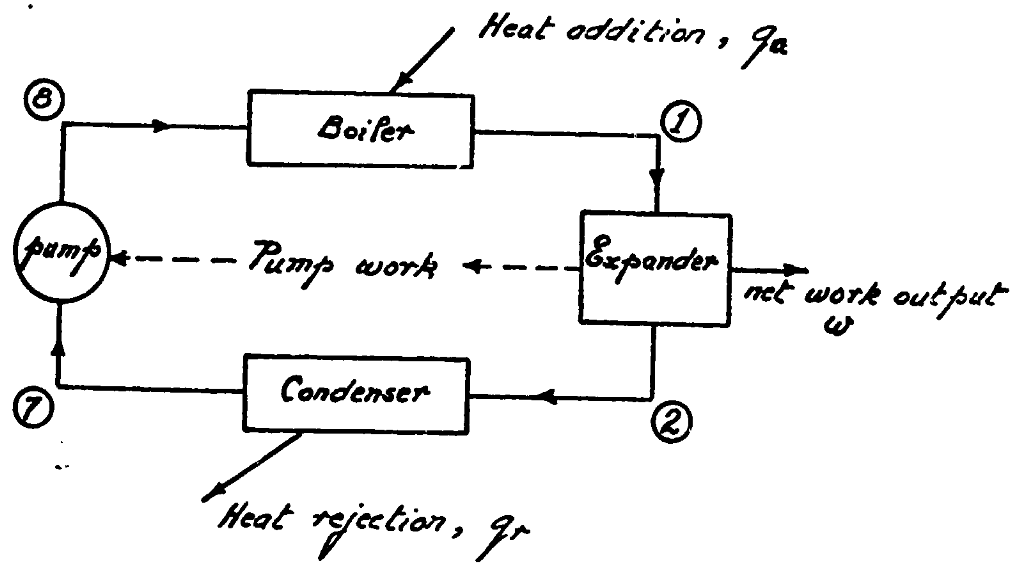


FIGURE 1

and transferred to a working fluid that is at high pressure. The fluid changes from liquid to vapor and then expands through either a turbine or a reciprocating expander to produce work.

The cycle is completed by condensing the fluid to liquid and pumping it back to the original pressure.

The use of a turbine under automotive size restrictions would require excessively high rotative speeds; thus, interest has centered primarily on the reciprocating expander. But, here too, there are serious problems. Chief among these is the development of efficient, reliable, long-life valves. Size restrictions prohibit low engine speeds at full power, and lubrication problems have not yet been solved. Development of a simple control system has also proven to be a formidable problem.

All this suggests investigating the use of fluidic controls and fluidic flow diverters in place of valves. Since there is little point in developing a control concept for a system that may not be feasible, major emphasis has been placed on a feasibility study of fluidic valving.

This study is divided into two parts. First, the thermodynamic implications are investigated, assuming that satisfactory flow diverters exist or that they can be developed. Second, a modest experimental evaluation of the control and response of a set of flow diverters is reported.

The most critical aspect of control for a Rankine cycle power plant is the boiler heat and feed pump control. A general evaluation of these control requirements is presented in Chapter IV. The report closes with conclusions and recommendations in Chapter V.

II. CYCLE ANALYSIS

Working Fluids

The Rankine cycle is shown on Temperature-entropy planes in Fig. 2 for three different fluids: water, CP-34, and Freon TF. The cycle assumes all processes to be reversible, i.e., ideal. Work is derived from the adiabatic expansion from state 1 to state 2, heat is rejected at constant pressure from state 2 to state 7, work is required to adiabatically increase the pressure from state 7 to state 8, and heat addition occurs at constant pressure from state 8 to state 1.

The thermal efficiency of the cycle is defined as the ratio of the net work output to the heat added. From the first law of thermodynamics the heat added can be expressed as the net work output plus the heat rejected. Thus,

$$\eta_t = \frac{w}{q_a} = \frac{w}{w + q_r} = \frac{1}{1 + \frac{q_r}{w}}$$

where η_t is thermal efficiency, w is net work output, q_a is heat added and q_r is heat rejected. Differences between kinetic energies at the various state points of the cycle are negligible. Therefore, energy transfers can be expressed in terms of enthalpy changes. In particular:

$$w = (h_1 - h_2) - (h_8 - h_7)$$
$$q_a = (h_1 - h_8)$$
$$q_r = (h_2 - h_7)$$

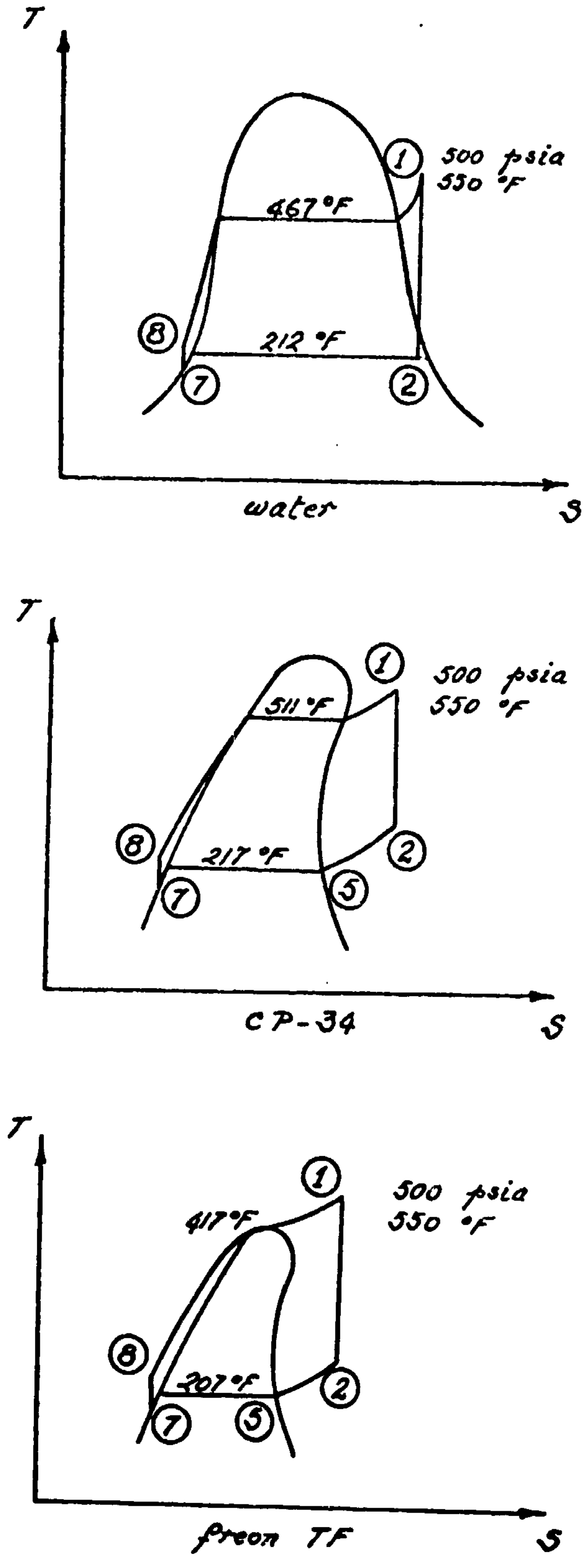

T
500 psia
550 °F
467 °F
212 °F
1
2
7
8
S
water
T
500 psia
550 °F
511 °F
217 °F
1
2
5
7
8
S
CP-34
T
500 psia
550 °F
417 °F
207 °F
1
2
5
7
8
S
Freon TF

Fig. (2)

It will be noted that at the end of the adiabatic expansion both CP-34 and Freon are still superheated. This makes it possible with these fluids to employ regenerative heating (Fig. 3).

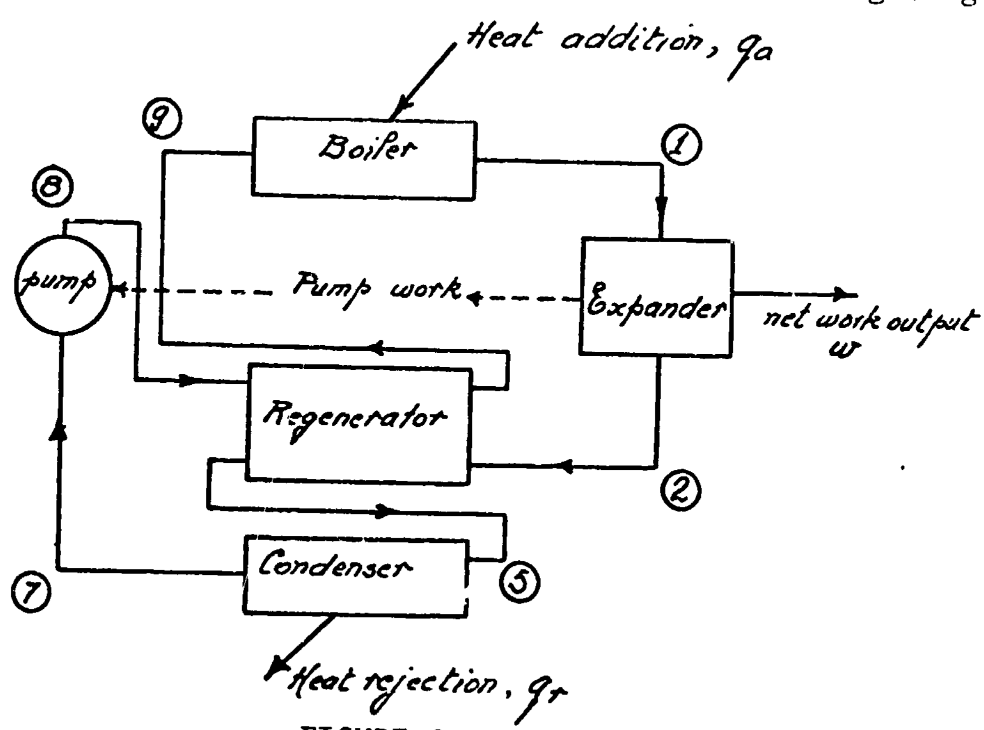


FIGURE 3

This reduces heat rejection and improves thermal efficiency. Assuming all the superheat is removed in an ideal regenerative process, the heat rejection for the cycle is $h_5 - h_7$.

The operating pressure of a condenser is determined by the temperature of the condensing fluid. The temperature is determined by the required heat transfer rate, the size and configuration of the condenser and the temperature of the cooling medium--air in this case. Because of size limitations in automotive applications, a rather substantial temperature difference between the condensing fluid and the ambient air must be anticipated for heavy loads. A subcooled condensate temperature of 200°F is assumed as a common base for all comparisons that are made in this report. Condensing temperatures in the range of 207°F to 217°F are chosen to correspond with convenient condenser pressures: 14.7 psia for water, 25 psia for CP-34 and 60 psia for Freon.

One more arbitrary assumption must be made before a comparison of performance with the three different fluids can be effected: state 1 must be selected. Other investigators working with CP-34 set 500 psia and 550°F as upper limits for state 1 to avoid the possibility of chemical decomposition. These values have been chosen for this study also. Whether they can be achieved safely with Freon TF has not been ascertained. It is understood that Freon decomposes into phosgene at high temperatures.

Data for comparing performance with the three fluids is given in Table 1. The thermal efficiency is highest for CP-34,

	w	q_r	η_t	$\rho_1 w$	ρ_1/ρ_2	$\rho_2 w$
	BTU/lb	BTU/lb	%	BTU/ft^3		BTU/ft^3
Water	268.5	828.	24.5	290.	20.8	13.9
CP-34	53.6	158.8	25.2	291.	18.0	16.2
Freon TF	17.7	57.	23.7	197.	6.1	32.3

TABLE 1 (ρ is density)

but the differences are not large. Differences in work per unit mass of fluid are enormous, but this primarily reflects large density differences. Work per unit volume of fluid is a more meaningful indicator of relative engine size. This parameter has different values at different state points and must be interpreted with care. At state 1 the work per unit volume is almost the same for water and CP-34, but for Freon it is much less. This indicates that the boiler, the flow diverters and the expander will have to be larger for Freon than for either water or CP-34 if the power ratings and engine speeds are to be the same. However, the isentropic expansion ratio (ρ_1/ρ_2) for Freon is much less than for either water or CP-34. Thus, the work per unit volume at discharge from the expander is much larger for Freon than for either water or CP-34. This indicates that the regenerator can be more compact for Freon than for CP-34, and it may be possible to make the condenser more compact as well.

The fact that the expansion ratio for Freon TF is much less than for the other fluids has another significant implication. Efficient use of large expansion ratios is not feasible with reciprocating expanders. The displacement volume would have to be excessive. To put it differently, the stroke would be large compared with the bore, and the speed would be relatively low. Instead, large irreversibilities are accepted in the "blow down" process. With Freon TF the reduction in thermal efficiency due to these irreversibilities should be less than with steam or CP-34.

Another significant characteristic of Freon TF is that its critical pressure is nearly equal to the maximum cycle pressure under consideration. By slightly increasing the cycle pressure, two-phase flow in the "boiler" can be avoided. This should simplify the problems of boiler control.

It is informative to compare the thermal efficiencies given in Table 1 with the Otto cycle thermal efficiency. The Otto cycle is the reversible model for spark ignited internal combustion engines just as the Rankine cycle is the reversible model for external combustion engines using condensable working fluids. For a compression ratio of 9:1 the air standard Otto cycle efficiency is 58.5%. This is more than twice the best efficiency given in Table 1. The reason, of course, is that temperatures are much higher in the Otto cycle than in the Rankine cycle. With CP-34 and Freon the temperature cannot be increased significantly because of the danger of chemical decomposition. With water the temperature can be increased, but safety and lubrication problems increase with it.

As can be seen in Fig. 1, steam expanding isentropically from state 1 penetrates well int the mixed phase region. Heat losses increase if condensation occurs on cylinder walls; and if the clearance volume is small, pressure relief may be required when the engine is cold. In the real cycle, however, penetration into the mixed phase region is reduced by throttling at the intake to the expander, and if the steam is superheated to high temperatures, the mixed phase region can be completely avoided in the expansion process.

One more observation should be made that is applicable to all three fluids under consideration. The saturation pressures corresponding to ambient temperature are below atmospheric pressure. This means that when the engine is shut down air can leak into the system unless the seals are perfect. The presence of air increases the condenser pressure and thus impairs performance. With steam it is not difficult to eject the air because steam can be bled to the atmosphere when the engine is operating. But this is not feasible with the other fluids. Furthermore, CP-34 and air react chemically, producing a contaminant. If Freon or CP-34 are to be used, perfect seals must be developed.

Each fluid has advantages and disadvantages. None is clearly superior, but in the remainder of this report only CP-34 is considered.

<u>Reciprocating Expanders</u>

In Fig. 4 an idealized pressure-volume diagram for a reciprocating expander is shown. It is drawn to scale for the case in which CP-34 is throttled from 500 psia and 550°F to 300 psia before entering the expander. At bottom dead center the piston uncovers the exhaust ports. The sequence of events depicted is as follows:

1. Constant pressure admission of the working fluid to the cylinder for a fraction A of the stroke. The fraction A is called the admission ratio and has the value of 1/2 in Fig. 4.

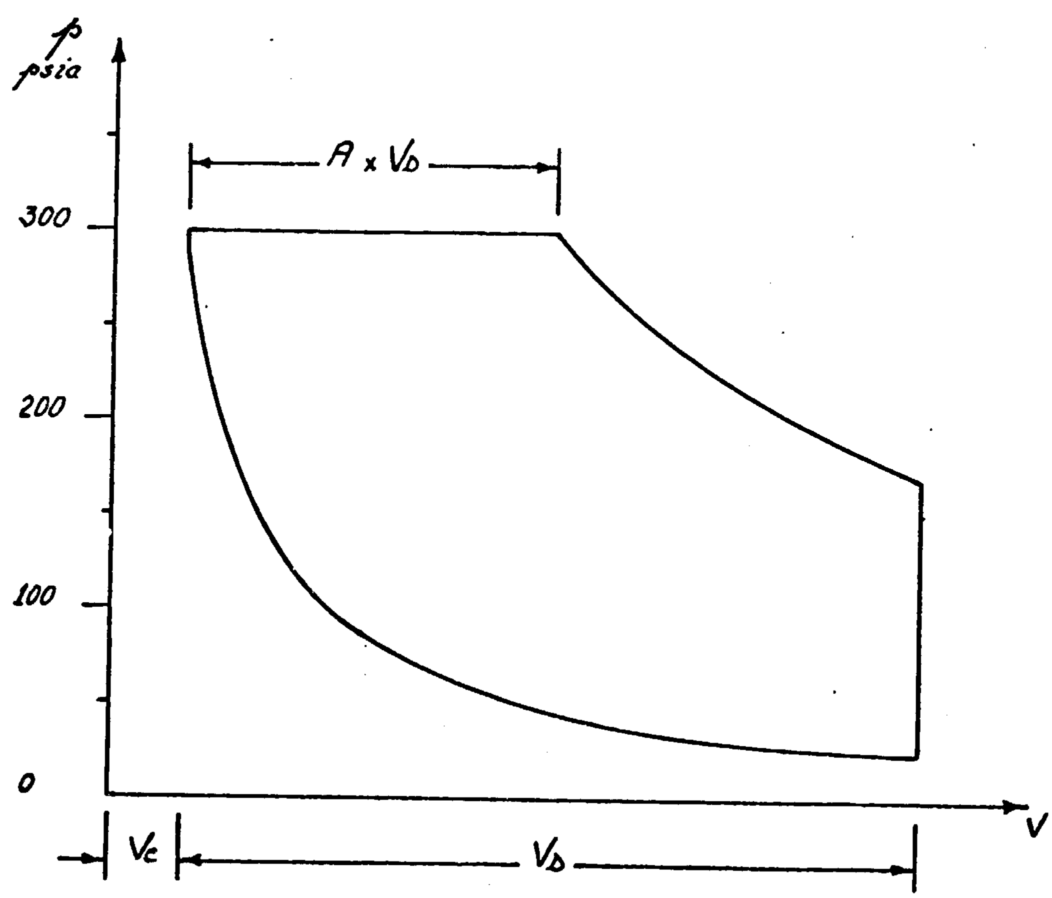


Fig. (4)

2. Isentropic expansion of the working fluid for the remainder of the stroke. The ratio of the cylinder volume at the beginning of this expansion to the cylinder volume at the end of the stroke is denoted by R and is called the cutoff ratio.

3. Irreversible "blow down" to the exhaust manifold pressure which is assumed to be the condenser pressure, i.e., 25 psia.

4. Isentropic compression of the residual fluid in the cylinder. The pressure at the end of compression depends on the ratio of the clearance volume to the displacement volume, and on the inlet state of the fluid. For $r = 0.1$, $p_1 = 300$ psia, and $h_1 = 122.14$ BTU/lb, the conditions depicted in Fig. 4, the pressure after compression is 287 psia. This is so nearly equal to p_1 that the irreversibility associated with the mismatch is negligible.

The area enclosed by the pressure-volume diagram divided by the displacement volume V_D gives the mean effective pressure. For this idealized case the mean effective pressure is 192 psi. The power output from the expander is given by the product of the mean effective pressure, the displacement volume and the engine speed. It is also given by the product of the mass flow rate of the working fluid and the difference in enthalpy between inlet and outlet of the expander if the process is adiabatic. This is expressed concisely by the following equation:

$$P = (\text{mep})\ V_D\ N = \dot{m}(h_1 - h_3) \tag{1}$$

The mass flow rate is equal to the mass in the cylinder at cutoff minus the mass in the cylinder after blow down all multiplied by the engine speed.

$$\dot{m} = [\rho_1 R V_D(1 + r) - \rho_2 V_D(1 + r)] N$$

$$= \rho_1 V_D N(R - \frac{\rho_2}{\rho_1})(1 + r) \quad (2)$$

It is convenient to define the volumetric efficiency of the engine as

$$\eta_v \triangleq \frac{\dot{m}}{\rho_1 V_D N} = (R - \frac{\rho_2}{\rho_1})(1 + r) \quad (3)$$

Equation (1) can now be rewritten as

$$P = (mep) V_D N = \eta_v \rho_1(h_1 - h_3) V_D N \quad (4)$$

from which

$$h_1 - h_3 = \frac{(mep)}{\rho_1 \eta_v} \quad (5)$$

To investigate the influence of the various design parameters on the performance of the engine it is desirable to have an equation for evaluating $h_1 - h_3$ that is not explicitly dependent on the mean effective pressure. Otherwise a very large number of pressure volume diagrams will have to be drawn to scale. Such an equation is derived in Appendix A; it is given here as equation (6).

$$h_1 - h_3 = h_1 - \frac{\rho_{2'} h_{2'} - \rho_2 h_2 - (p_{2'} - p_2)}{\rho_1(R - \frac{\rho_2}{\rho_1})} \quad (6)$$

The thermodynamic processes associated with the expander are shown on an enthalpy-entropy plane in Fig. 5. State 1 is the inlet state to the expander (after throttling, if any). State 2' is determined by isentropic expansion from state 1

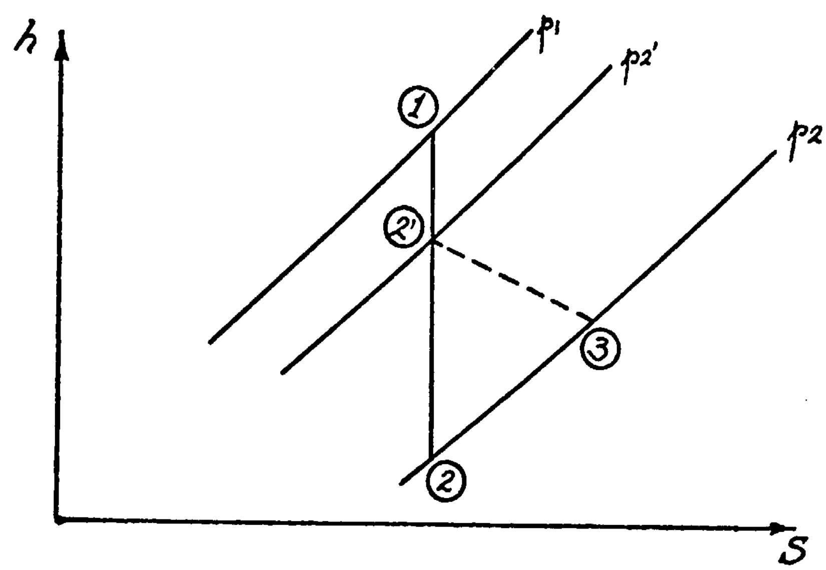
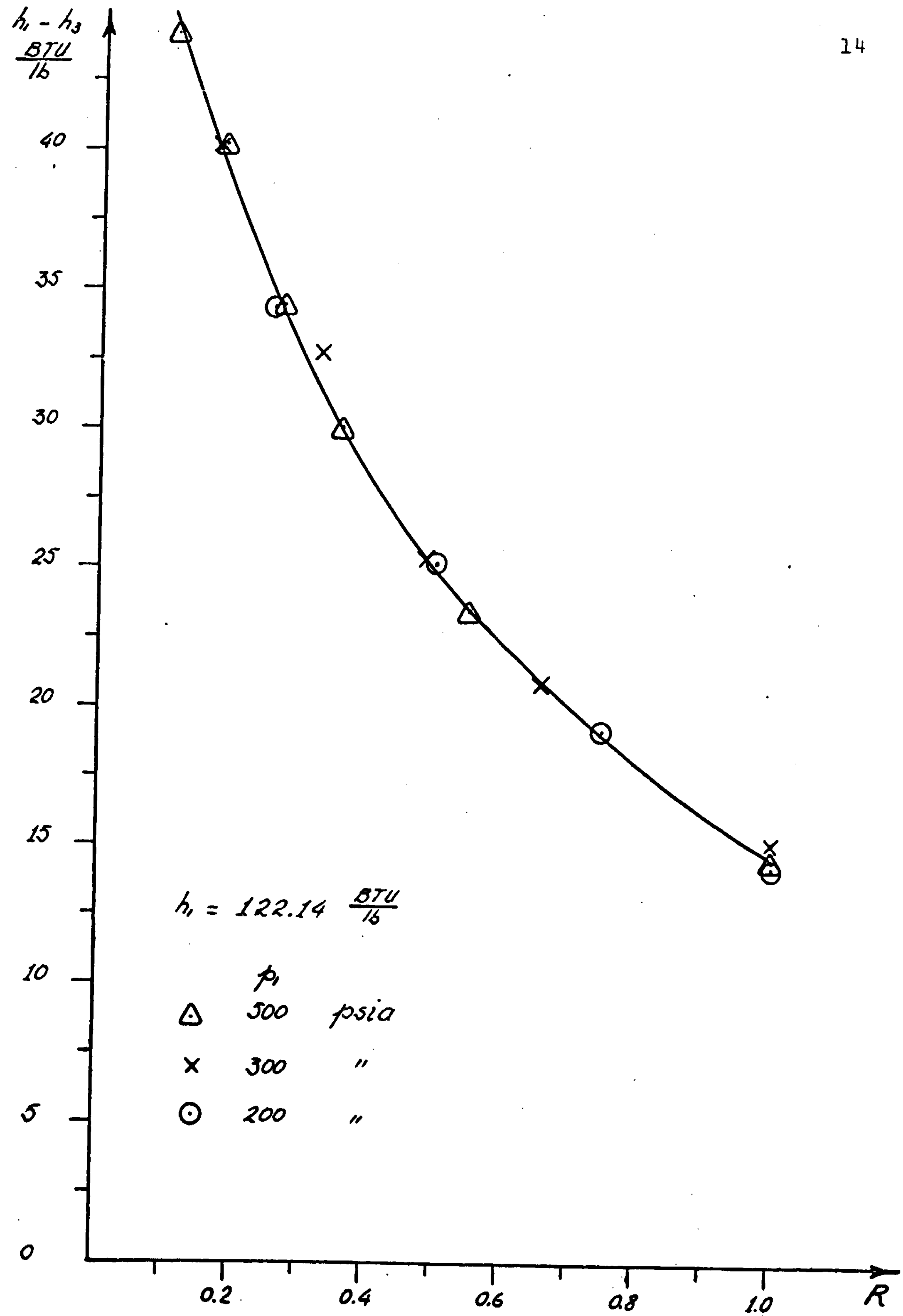


FIGURE 5

through the expansion ratio R^{-1}. State 2 is the state of the residual fluid in the cylinder after blow down. It is determined by isentropic expansion from state 1 to the exhaust manifold pressure. State 3 is the state of the fluid in the exhaust manifold after the turbulent mixing of the blow down process.

In Fig. 6 the effect of the cutoff ratio on $h_1 - h_3$ is shown for three different values of p_1. Computations for this curve were made early in the study before computational techniques were refined. Some values may be in error by as much as 1 BTU/lbm, but the overall conclusions are nevertheless inescapable:

1. In view of Eq. (1) and Fig. 6, the cutoff ratio should be as small as possible to achieve a minimum "steam" generation rate for a given power output.
2. For a fixed inlet enthalpy the inlet pressure has very little effect on the work per pound of fluid. However, it does influence the power through its influence on the density at state 1 (Eq. (4)).



$h_1 - h_3$
BTU/lb
40
35
30
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1.0
R
h_1 = 122.14 BTU/lb
p_1
500 psia
300 "
200 "

Fig. (6)

Using values from the curve of Fig. 6 to compute the net work output for the cycle ($w = (h_1 - h_3) - (h_8 - h_7)$), values of thermal efficiency for the cycle were computed. They are shown in Fig. 7. Also, the effect of admission ratio and clearance volume ratio is shown. The latter is merely a graph of the geometrical relationship

$$R = \frac{1 + A}{1 + r}$$

Clearly, a small admission ratio and a small clearance volume ratio are highly desirable.

It is apparent from Eq. (3) that the volumetric efficiency decreases linearly with decreasing values of R if the inlet state is held constant. Indeed, the percentage change in η_v is much greater than the percentage change in $(h_1 - h_3)$; thus, as seen from Eqs. (4) and (5), the power and mean effective pressure decrease with decreasing values of R. If the cutoff ratio is to be small at the design point, then either the displacement volume or the speed must be large. Automotive applications preclude large displacement volumes, and valve dynamics preclude the combination of low admission ratio and high speed.

What are the implications of all this for design and control? First, the cutoff ratio at the design point should be made as small as practicable. Some other investigators working with CP-34 are using values of R approximately equal to one-tenth. It is the opinion of this investigator that this is too

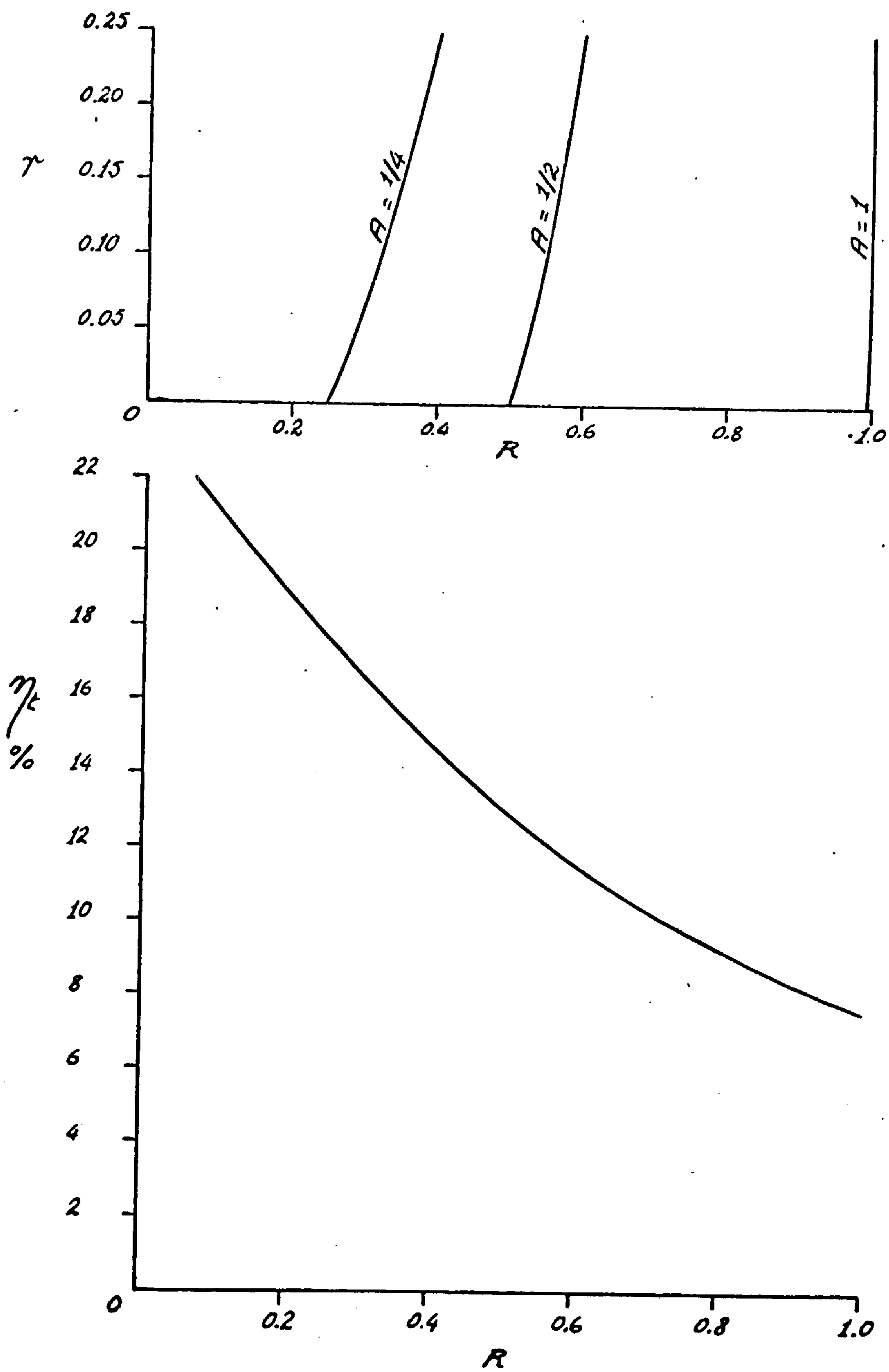

0.25
0.20
0.15
0.10
0.05
0
r
A = 1/4
A = 1/2
A = 1
0.2
0.4
0.6
0.8
1.0
R
22
20
18
16
14
12
10
8
6
4
2
0
η_t %
0.2
0.4
0.6
0.8
1.0
R

Fig. (7)

low for the present state of the art. Second, for high-speed conditions, i.e., reduced torque but approximately design speed, throttle control should be employed. For low speed cruising, variable admission control would be desirable. For low-speed high torque requirements, variable admission control would be desirable if one is willing to sacrifice efficiency to eliminate a transmission. A control system that incorporates both throttle control and variable admission control would be ideal; but, if only one method can be employed, it should be throttle control.

Fluidic Flow Diverters

If fluidic flow diverters are used in place of valves, continuous flow of the working fluid must be maintained. This fact sets minimum values for the admission ratio as a function of the number of cylinders. For two cylinders $A = 1$; for four cylinders $A = 1/2$ and two stages of flow diverters are required; for eight cylinders $A = 1/4$ and three stages are required. Of these possibilities the most feasible appears to be the four cylinder arrangement. Therefore, the remainder of the cycle analysis will be for $A = 1/2$; with $r = 0.1$, $R = 0.546$.

The important performance parameters are given in Table 2. In interpreting these values it is important to bear in mind that computed values are based on a highly idealized pressure-volume diagram for the expander, on a highly effective regenerator, and on all other processes in the cycle being reversible. Nevertheless, there is a substantial reduction in the thermal efficiency from that of the Rankine cycle.

h_1 BTU/lb	P_1 psia	ρ_1 lb/ft^3	Δh_{1-3} BTU/lb	η_s %	η_v %	η_t %
122.1	500	5.43	24.3	44.1	54.7	12.5
"	300	2.90	24.7	44.8	50.4	12.7
"	200	1.81	23.6	42.9	45.2	12.2
"	100	0.87	20.3	36.8	30.7	10.7
111.6	500	5.88	22.3	42.0	54.9	11.6
"	300	3.08	23.4	44.1	50.7	12.1
"	200	1.90	22.9	43.1	45.2	11.9

TABLE 2

The isentropic efficiency η_s is the ratio of the work output of the expander to the work that could be derived from an isentropic expansion from the boiler exit state to the condenser pressure. Its nearly constant value over the upper range of inlet pressure indicates that the trade-off of irreversibility between throttling at inlet and free expansion at blow down has little effect on efficiency.

The enthalpy of 122.1 BTU/lb corresponds to a boiler exit state at 500 psia and 550°F. This amounts to 38.6°F superheat. The enthalpy of 111.6 BTU/lb corresponds to a boiler exit state at 500 psia and 525°F or 13.6°F superheat. The effect of superheat on expander performance appears to be rather modest.

The lower pressure states in Table 2 could be achieved by letting the boiler pressure decrease rather than by throttling. But, if this were contemplated as another method of control, it should be abandoned immediately. The (122.1 BTU/lb, 300 psi) state requires twice as much superheat at 300 psia boiler pressure as it does at 500 psia. The boiler design cannot tolerate this sort of variation and the situation would be even worse at lower pressures. We conclude that boiler pressure should be held as constant as possible.

The use of fluidic flow diverters will necessitate a substantial pressure drop between boiler and expander even at wide open throttle. Fortunately, this does not significantly influence thermal efficiency. The pressure drop to expect is difficult to judge, for appropriate diverters have yet to be developed. A drop from 500 psia to 300 psia has been assumed. If this should prove to be excessive, the displacement volume that is calculated will be too large, but there will be no other significant effect. For design point calculations, then, an inlet state to the expander of h_1 = 122.1 Btu/lb and p_1 = 300 psia is assumed.

It is now necessary to estimate factors by which the idealized performance parameters can be converted to real performance parameters. In doing so, values equal to those employed by other investigators will usually be used even though this investigator is inclined to be more pessimistic.

First, a factor of 0.846 will be assumed to convert the idealized mean effective pressure to an "indicated" mean effective pressure. This factor, expressed in per cent, will be referred to as the "diagram efficiency"*. To maintain the equality of

*Other investigators have called this the "engine thermal efficiency". In the opinion of this investigator that is poor terminology. Thermal efficiecies should be restricted to cycle performance.

Eq. (5) when the diagram efficiency factor is applied, a factor must also be applied to either η_v or $h_1 - h_3$ or both. There are two effects to be considered in going from the idealized pressure-volume diagram to the true indicator diagram: the effective cut-off ratio is reduced, and the process is no longer adiabatic. These two effects have opposing influences on the indicated work per unit mass of fluid. It would require a highly refined analysis to determine the net effect; so the assumption is made that there is none: the indicated work per unit mass is assumed to be equal to $h_1 - h_3$ as computed from Eq. (6). The entire diagram efficiency factor is therefore applied to the volumetric efficiency. At the design point $\eta_v = 0.846 \times 0.504 = 42.6\%$.

The mechanical efficiency η_m is defined as the ratio of the brake mean effective pressure to the indicated mean effective pressure. It is also the ratio of brake power to indicated power and of brake work to indicated work. The brake work of the expander is $\eta_m(h_1 - h_3)$; and if $\eta_m = 91.5\%$, the brake work for the design point is 22.6 BTU/lb.*

The feed pump overall efficiency η_p can be defined as the ratio of the isentropic pump work to the actual pump work. Thus, the actual pump work is $(h_8 - h_7)/\eta_p$. For $\eta_p = 79.7\%$, the pump work is (1.46 BTU/lb)/0.797 or 1.83 BTU/lb.

The net work output for the cycle is given by

$$w = (h_1 - h_3)\,\eta_m - \frac{(h_8 - h_7)}{\eta_p}$$

$$= 22.6 - 1.8 = 20.8 \text{ BTU/lb}$$

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*Other investigators have reported an "engine overall efficiency". It is the product of the diagram efficiency and the mechanical efficiency.

If the regenerator was able to extract all of the superheat from the expander exhaust, the heat rejection at the condenser would be $h_5 - h_7 = 158.8$ BTU/lb. Then, $\frac{q_r}{w} = 7.64$ and $\eta_t = (1 + \frac{q_r}{w})^{-1} = 11.6\%$. But to do this the regenerator would have to have a 93.6% effectiveness and an N.T.U. value of 6.72.* This indicates a large heat transfer area. Other investigators have assumed 90% effectiveness, but this seems unreasonably high for an automotive application; 75% would be more rational. With this latter value, the state at entrance to the condenser would have 11.4 BTU/lb superheat, the heat rejection would be 170.2 BTU/lb, and the thermal efficiency of the cycle would be 10.9%.

The cycle thermal efficiency is based on the heat added to the working fluid. The overall efficiency, however, is based on the energy supplied by the fuel. This is introduced by defining a boiler efficiency η_b.

$$\eta_b \triangleq \frac{\text{heat added to working fluid}}{\text{energy supplied by fuel}}$$

Assuming $\eta_b = 82.5\%$, the overall efficiency becomes

$$\eta = \eta_t \, \eta_b = 9.0\%$$

The overall efficiency of an automotive internal combustion engine is 2-1/2 - 3 times this value. It follows that the fuel consumption of the power plant under consideration would be 2-1/2 - 3 times that of a comparable internal combustion engine.

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*N.T.U. = $AU/\dot{m}c)_{min}$, where A is the heat transfer area, U is the overall heat transfer coefficient, $\dot{m}$ is the mass flow rate and c is specific heat.

Engine Size and Speed

The power developed by the cycle is given by

$$P = \dot{m}w$$

If this power is specified as 100 hp, then the brake power of the expander will be 108.5 hp and the feed pump power will be 8.65 hp. However, if bypass control is used for the pump, its power consumption will be increased to about 10 hp. The mass flow rate of working fluid will be 3.40 lb/sec except through the pump. At the pump there will be 3.93 lb/sec or 29.1 gal/min.

With a volumetric efficiency of 42.6% and an engine speed of 2000 rpm the displacement volume is found to be 143 in^3. This can be achieved with four cylinders having a 3.89" bore and a 3.00" stroke. This gives a mean piston speed of 1000 ft/min and a maximum speed of 3142 ft/sec. At this speed the second stage flow diverters would have to switch and dwell for 0.0075 sec. The portion of this time that is devoted to switching should be as small as possible, i.e., a square wave pulse would be ideal. If a reasonable approximation to this cannot be achieved, the speed should be reduced and the displacement volume increased proportionately.

Switching times in the order of one millisecond can be achieved with small fluidic devices, but the switching time undoubtedly increases with size. If this be the case, it is difficult to see any hope for the use of fluidic flow diverters in place of mechanical valves.

One other factor unique to fluidic flow diverters must be taken into account: they must be vented to an ambient pressure below cylinder pressure. At one time it was proposed that this ambient pressure should be intermediate between boiler pressure and condenser pressure and that flow from the region should be used to power auxiliary equipment. However, it is necessary to use fluidic diodes between the cylinders and diverters, and for these to function properly the cylinder pressure must always be greater than the vent pressure. The vent pressure must therefore be the condenser pressure and no auxiliary power can be derived from vent flow. Indeed, the vent flow is a complete loss; and, if it amounts to 10% of the expander flow, the overall efficiency will be reduced from 9.0% to 8.2%. Since the amount of vent flow is unknown, its effect will not be accounted for in the remainder of this report; but it should be noted that it influences not only the overall efficiency but also the size of the boiler, condenser and feed pump.

For design point operation, the boiler must have a heat transfer rate of 2.34×10^6 BTU/hr. This is approximately 1-1/2 times larger than the rate proposed by other investigators. Similarly, the heat transfer rate at the condenser is 2.09×10^6 BTU/hr, 1-2/3 times the value proposed by others. Space requirements will be correspondingly greater.

Auxiliary Power Requirements

If a fuel with a higher heating value of 18,800 BTU/lb is assumed, a fuel flow rate of 0.0418 lb/sec is required at design point conditions. Assuming a specific gravity of 0.8 and a nozzle pressure of 500 psi, 0.110 hp would be required with a 100% efficient fuel pump.

With 25% excess air, 0.784 lb/sec or 628 scfm combustion air flow would be required. With a draft of 1.5 inches of water, 0.148 hp is needed. Allowing for reasonable fuel pump and fan efficiencies, about 1/2 hp is required to supply the boiler with fuel and air.

At the condenser, however, the air requirements are a different order of magnitude. Assuming a 20°F rise in temperature an air flow rate of 120 lb/sec or 96,000 scfm is required. If a pressure rise of 5 lb/sq ft across the fans is assumed, then 14.5 hp is required. With a fan efficiency of 74.6%,19.5 hp is needed. Thus, 20 hp is required to operate the engine at design point conditions and only 80 hp is left for operation of the vehicle and other accessories, such as an alternator, hydraulic pump and air conditioner. This latter group of accessories may require another 15 hp.

Summary of Design Point Data

Working Fluid	CP-34
Boiler Outlet Pressure	500 psia
Boiler Outlet Temperature	550°F
Boiler Heat Transfer Rate	2.34×10^6 BTU/hr
Boiler Efficiency (HHV)	82.5%
Brake Power Less Feed Pump Power	100 hp
Brake Mean Effective Pressure	151 psi
Engine Speed	2000 rpm
Displacement Volume (4 cylinders - 3.89" x 3.00")	143 in^3
Volumetric Efficiency	42.6%
Indicated Isentropic Efficiency	44.8%
Mechanical Efficiency	91.5%
Regenerator Effectiveness	75%
Regenerator Heat Transfer Rate	0.38×10^6 BTU/hr
Condenser Pressure	25 psia
Subcooled Liquid Temperature	200°F
Condenser Heat Transfer Rate	2.09×10^6 BTU/hr
Mass Flow Rate	3.40 lb/sec
Volumetric Flow Rate (at pump)	25.2 gal/min
Pump Power	8.65 hp
Pump Efficiency	79.7%
Overall Efficiency	9.0%

III. EXPERIMENTAL EVALUATION OF FLUIDIC FLOW DIVERTER CONTROL SCHEME

Fluidic flow diverters are conceived to be bistable fluid amplifiers designed for high level power transmission rather than the mere processing of low level signals. Several schemes for generating control signals were considered, and it was decided to experimentally check the workability of two of these.

Three Norgren flip-flops were connected, as shown in Fig. 1 (of this chapter), and loaded by four "visiwinks". Control signals were generated by a rotating arm that interrupted four jets (Fig. 2). This scheme worked, but it was very sensitive to the clearance between the arm and the tips of the jets. In actual application, the rotating arm could be a projection on the crankshaft of the engine, and the set of jets could be rotated to adjust the timing. By turning the engine over with an electrically driven starter, proper sequencing would be established.

One disadvantage of this scheme would be the increased "steam" rate required to supply virtually continuous flow to four jets. An alternate scheme was investigated in which control signals would be generated by relieving normally blocked ports. The rotating arm was replaced by a disk with a hole in it (Fig. 3). The flow from the jets was thus greatly reduced except when aligned with the hole.

This scheme did not work. Perhaps a circular arc slot should have been used instead of a hole. Perhaps impedance

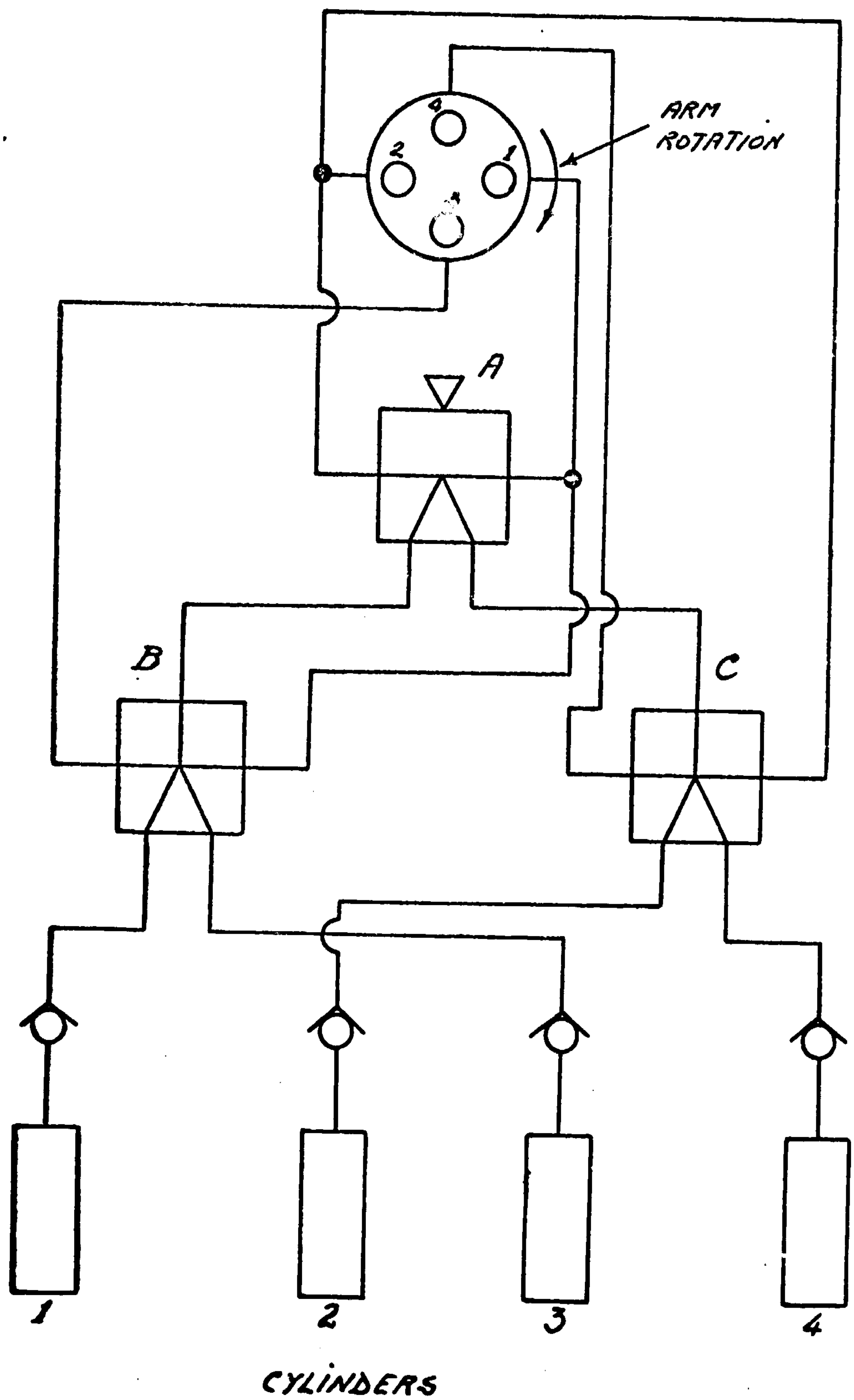
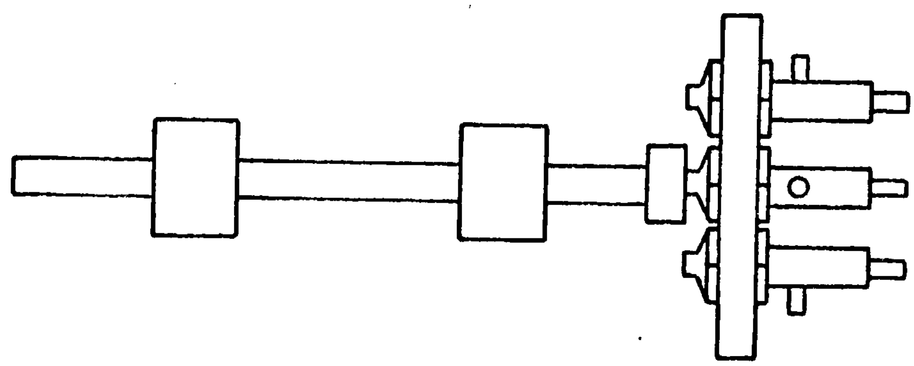
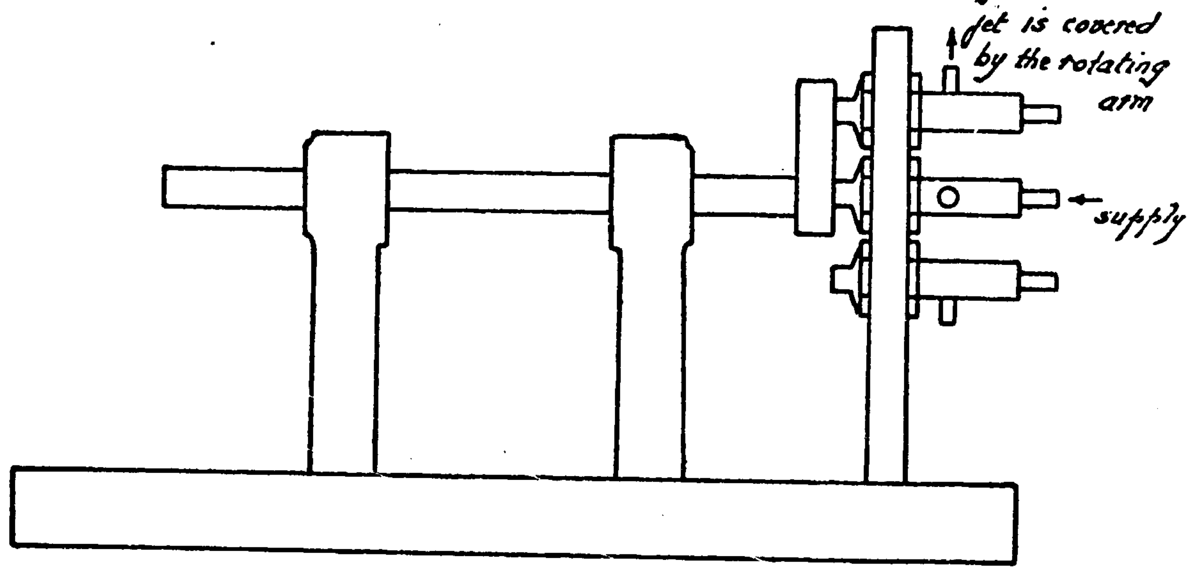


Fig. (1)



PLAN



ELEVATION

Fig. (2)

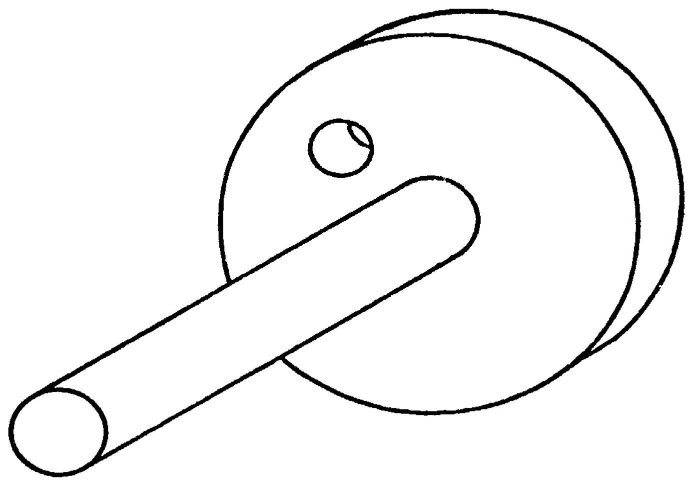


Fig. (3)

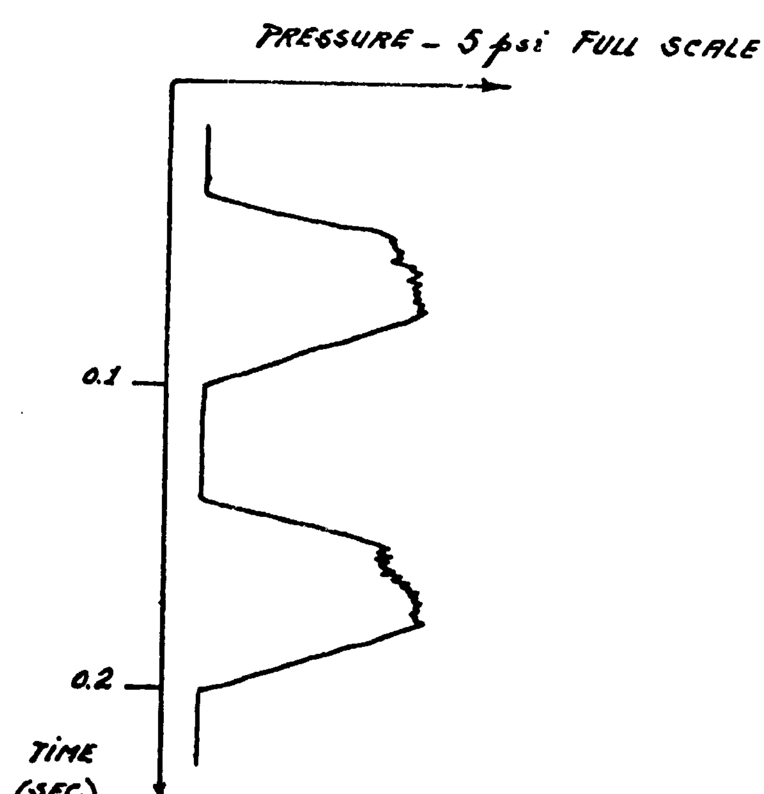


Fig. (4)

matching in the control lines is critical. These questions were not resolved.

A differential pressure transducer was used to measure the output of flip-flop B (Fig. 1) as a function of time. The result is shown in Fig. 4. The output does approximate a square wave, but the period is very large. This is because the arm was rotated by hand. When an attempt was made to drive it with an electric motor, the system did not operate. Probably the test fixture was not rigid enough to maintain proper clearance between the nozzles and the rotating arm under the dynamic loading of imbalance.

The output lines from the flip-flops were relatively long and small in bore compared with those that would be used in an actual application. It is believed that this significantly influenced the rate of pressure rise in the output signal.

All the difficulties encountered in this test program can be attributed to the test facilities rather than the basic concept. The tests neither prove nor disprove the control schemes. They do point to areas in which development problems should be anticipated.

IV. SUMMARY OF CONTROL SYSTEM ANALYSIS

A detailed analysis of the boiler control system is described in Appendix C. A summary of the most important results is presented here.

A feasible control law is described by the following equations:

$$\frac{dQ}{dt} = a_{11}(V_f - \overline{V}_f) + \beta \dot{m}_1 ,$$

$$K_7 = a_{22}(p_o - \overline{p}_o) + g(\dot{m}_1, N)$$

where $\frac{dQ}{dt}$ is the rate of heat transfer to the boiler,

k_7 is the feedwater pump bypass valve gain,

V_f, $\overline{V}_f$ are the actual and desired values, respectively, of the boiler liquid level,

p_o, $\overline{p}_o$ are the actual and desired values, respectively, of boiler pressure,

a_{11}, a_{22}, and β are constants, and

$g(\dot{m}_1, N)$ is a function which is part of the controller design.

The appendix considers other, somewhat more general control laws which are also possible. Note also that the essential effect of the two terms involving the constants a_{11} and a_{22} could be produced (more realistically) by two nonlinear functions

$$f_{11}(V_f - \overline{V}_f) \text{ and } f_{22}(p_o - \overline{p}_o)$$

In this latter case, the analysis of performance would be rather more difficult than that presented in the appendix and would most appropriately be carried out by computer.

Performance of the control system is separated naturally into two aspects: steady-state error and transient response. The steady-state errors are given by:

$$\tilde{V}_f - \bar{V}_f = \left(\frac{\Delta h_{9-1} - \beta}{a_{11}}\right) \dot{m}_1$$

and $$\tilde{p}_o - \bar{p}_o = \frac{1}{a_{22}} [\hat{f}(\dot{m}_1, N) - g(\dot{m}_1, N)],$$

where the ~ refers to actual steady-state values, and the function $\hat{f}$ is defined in Appendix C. The errors may be made zero (or small) through the choice of β, $g(\cdot)$, a_{11} and a_{22}. There are two basic strategies for reducing the errors:

(1) Approximate as nearly as possible the conditions

$$\beta = \Delta h_{9-1}$$

and $$g(\dot{m}_1, N) = \hat{f}(\dot{m}_1, N),$$

and (2) Make the magnitudes of a_{11} and a_{22} as large as possible, consistent with transient behavior (to be discussed later) and practical constraints on implementation.

Neither of the conditions in (1) can be achieved exactly in practice. For example, Δh_{9-1} depends to some extent on engine operating conditions; hence, a constant value of β cannot satisfy the equality. Therefore, a combination of both strategies should be used.

The transient behavior to a first approximation is described by a second-order characteristic equation (Eq.(22) in Appendix C). The coefficients of the characteristic equation are functions of a_{11} and a_{22}; hence, the values of the characteristic roots may be set by appropriate choices of a_{11} and a_{22}. There are standard control system criteria for adjusting the root locations, based for instance on desired percentage overshoot and settling time. However, at this time, without more knowledge of actual system parameters, there is little value in pursuing such a detailed analysis further.

A realistic controls design should consider the following factors:

1. The possible interdependence between steady-state errors and transient performance through the values of a_{11} and a_{22}.
2. The variation of most system parameters with operating conditions. The design must be carried out so as to insure both satisfactory errors and transient response over the full range of operating conditions.
3. Nonlinearities. The only practical way to take accurate account of system nonlinearities will be computer simulation, which would be premature at this stage of development before selection of specific hardware.

V. CONCLUSIONS AND RECOMMENDATIONS

If further work is to be done in exploring the possibility of using fluidic flow diverters in place of mechanical valves for a reciprocating expander, a number of fundamental questions must be answered:

1. What is the relationship between pressure drop and flow rate for a fluidic flow diverter and diode operating with an inlet pressure of 500 psia and with flows up to 4 lb/sec?
2. How closely can the output approximate a square wave and what is the minimum value of its period?
3. What per cent of the flow passes out the vent?

However, even if favorable answers are found to these questions, the overall efficiency will remain low because the admission ratio cannot be made small. It is therefore recommended that no further work be done in developing fluidic flow diverters for use with reciprocating expanders.

If further work is to be done in applying fluidics to Rankine cycle engines, it should be done in the area of information sensing and processing in the control of existing engines. This, of course, would have to be done in cooperation with the developers of these engines, for detailed information on component performance is required.

But here, too, the outlook is not very encouraging. Unless low admission ratios and high cycle temperatures can be achieved, Rankine cycle engines are bound to have much lower efficiencies than internal combustion engines. One might argue that people will be willing to pay higher transportation costs in order to reduce air pollution. But the problem is that lower efficiency means higher fuel consumption; lower concentrations of pollutants in the exhaust do not necessarily mean less pollution. For 9% overall efficiency (the best that can be expected for the system of this study) the 1980 air pollution standards would be barely satisfied. There is little doubt but what the internal combustion engine can be made to satisfy these standards and at much better than 9% efficiency.

There is also growing concern for the rate at which the world's energy resources are being consumed. The automobile accounts for a major fraction of the energy used in the United States, and higher automotive fuel rates would significantly influence the energy consumption in this country. In attempting to solve one problem we must be careful not to create or intensify other problems.

APPENDICES

APPENDIX A

DETERMINATION OF ENTHALPY IN THE EXPANDER EXHAUST

Consider the vapor in a cylinder immediately before blow down to be a closed system. Its volume is $(1 + r)\ V_D$; its mass is $(1 + r)\ V_D\ \rho_{2'}$. The internal energy U_i is $(1 + r)\ V_D\ \rho_{2'}$ $(h_{2'} - \frac{p_{2'}}{\rho_{2'}})$.

During blow down the vapor expands isentropically to pressure p_2. No work is done on the piston, for it is at rest at bottom dead center. However, the vapor leaving the cylinder does work on the vapor already in the exhaust manifold as it pushes it back against the uniform pressure p_2. The fluid leaving the cylinder has kinetic energy that is transformed through turbulence into internal energy, thus producing state 3.

The mass remaining in the cylinder after blow down is $(1 + r)\ V_D\ \rho_2$; thus, the mass leaving the cylinder is $(1 + r)\ V_D$ $(\rho_{2'} - \rho_2)$ and the change in volume of the system ΔV is $(1 + r)$ $V_D\ (\frac{\rho_{2'} - \rho_2}{\rho_3})$. The final internal energy U_f of the system is $(1 + r)\ V_D\ [\rho_2\ (h_2 - \frac{p_2}{\rho_2}) + (\rho_{2'} - \rho_2)(h_3 - \frac{p_2}{\rho_3})]$.

According to the first law of thermodynamics for an adiabatic process,

$$U_f - U_i = - p_3\ \Delta V$$

Substituting the above expressions into this equation and solving for h_3 gives:

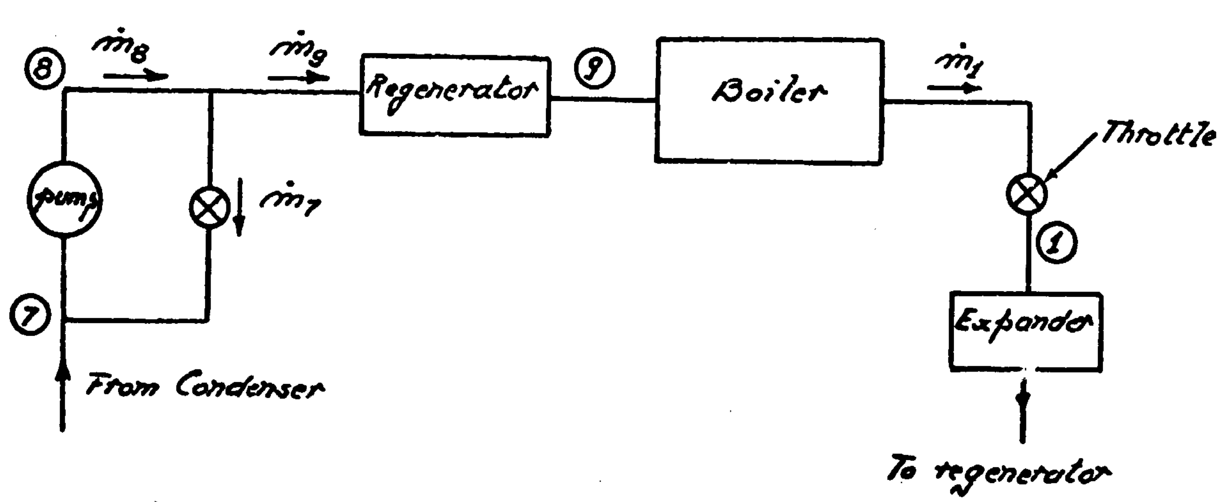
$$h_3 = \frac{\rho_{2'}\ h_{2'} - \rho_2\ h_2 - (p_{2'} - p_2)}{\rho_{2'} - \rho_2}$$

For convenience $\rho_{2'} - \rho_2$ is replaced by $\rho_1(R - \frac{\rho_2}{\rho_1})$ since $R = \frac{\rho_{2'}}{\rho_1}$.

APPENDIX B
BOILER AND FEED PUMP CONTROL MODEL

It is clear from the information presented in Chapter II that over a wide range of engine operating conditions the efficiency is approximately constant. This, together with the fact that the efficiency is low, means that the steady-state heat transfer rate at the boiler should be proportional to the mass flow rate of the working fluid, and Δh_{9-1}, the change of enthalpy through the boiler, should be essentially constant.

The steady-state mass flow rate is determined by the engine speed and throttle setting. If the feed pump has a fixed positive displacement and is driven directly by the expander, then a pump bypass valve must be employed to compensate for throttle setting and transient effects. This is the system depicted in the figure below. A small electrically driven pump (not shown) can be used for startup.



In order to analyze transient behavior, the following assumptions are made:

1. The state at inlet to the feed pump is constant.
2. The pump flow $\dot{m}_8$ is approximately proportional to speed N.
3. The boiler is divided into three stages. The phase change occurs within the second stage where the pressure is p_o. The total volume of the second stage is denoted by V, the volume of liquid by V_f and the volume of vapor by V_g.
4. The pressure drop across the bypass valve is given by:

$$p_8 - p_7 = k_7 \; v_7 \; \dot{m}_7^2$$

where k_7 is determined by the bypass valve setting and v is specific volume (at state 7 in this case).

5. The pressure drop between the pump and the second stage of the boiler is given by:

$$p_8 - p_o = k_9 \; v_9 \; m_9^2$$

6. The pressure drop between the second stage of the boiler and the cylinders of the expander is given by:

$$p_o - p_1 = k_1 \; v_1 \; \dot{m}_1^2$$

where k_1 is determined by the throttle setting.

From continuity it is clear that

$$\dot{m}_9 = \dot{m}_8 - \dot{m}_7$$

when this is combined with assumptions 4 and 5, the ratio of the flow into the boiler to the flow from the pump can be determined.

$$\frac{m_9}{m_8} = \frac{1 - \sqrt{1 - \left(1 - \frac{p_o - p_7}{k_7 \; v_7 \; m_8^2}\right)\left(1 - \frac{k_9 \; v_9}{k_7 \; v_7}\right.}}{1 - \frac{k_9 \; v_9}{k_7 \; v_7}}$$

Since $\dot{m}_8$ is a function of speed,

$$\dot{m}_9 = f(p_o, k_7, N)$$

During a transient period the flows into and out of the boiler need not be the same. Their difference is denoted by $\Delta\dot{m} \triangleq \dot{m}_1 - \dot{m}_9$. The difference is accounted for by changes in the proportion of liquid and vapor in the second stage of the boiler and by changes of specific volume if the boiler pressure changes. This is expressed mathematically as:

$$\Delta\dot{m} = f_1(p_o)\frac{dp_o}{dt} - f_2(p_o)\frac{dV_f}{dt}$$

The derivatives are with respect to time, and

$$f_1(p_o) = \frac{V_f}{v_f^2}\frac{dv_f}{dp_o} + \frac{V_g}{v_g^2}\frac{dv_g}{dp_o}$$

and $f_2(p_o) = \frac{1}{v_f} - \frac{1}{v_g}$

The specific volumes v_f and v_g are for saturated liquid and saturated vapor respectively and are functions of p_o.

The first law of thermodynamics can be written for the boiler in the following form:

$$\frac{dQ}{dt} + \dot{m}_9 h_9 - \dot{m}_1 h_1 = \frac{dE}{dt}$$

$\frac{dQ}{dt}$ is the heat transfer rate and $\frac{dE}{dt}$ is the rate of change of energy within the boiler. For steady-state conditions, $\frac{dE}{dt}$ is zero, but during a transient it has a value because of changing proportions of liquid and vapor as well as changing temperature and pressure.

When these terms are evaluated, the above equation can be expressed as:

$$\frac{dQ}{dt} - \dot{m}_g \, \Delta h_{g-1} - \Delta\dot{m} \, h_1 = (\overline{mc} \, \frac{dT_o}{dp_o}) \frac{dp_o}{dt} + F_1(p_o) \frac{dV_f}{dt} + F_2(p_o, V_f) \frac{dp_o}{dt}$$

where $\overline{mc}$ is the mean heat capacity of the boiler excluding the fluid within the second stage,

$$F_1(p_o) = \frac{h_f}{v_f} - \frac{h_g}{v_g}$$

and

$$F_2(p_o, V_f) = \frac{V_f}{v_f} \frac{dh_f}{dp_o} + \frac{V_g}{v_g} \frac{dh_g}{dp_o} - h_f \frac{V_f}{v_f^2} \frac{dv_f}{dp_o} - h_g \frac{V_g}{v_g^2} \frac{dv_g}{dp_o} - V$$

It should be noted that the term $(\overline{mc} \, \frac{dT_o}{dp_o}) \frac{dp_o}{dt}$ is a gross oversimplification. The term should be a function of the temperature distribution throughout the boiler and its rate of change. In the equation for $\Delta\dot{m}$ there is also an oversimplification, for changes of specific volume occur in all stages of the boiler, not just the second stage; but these changes, too, would depend on the temperature distribution throughout the boiler.

APPENDIX C
CONTROL SYSTEM ANALYSIS

The dynamic behavior of the boiler system is described by the set of equations

$$\Delta\dot{m} = f_1(p_o)\,\frac{dp_o}{dt} - f_2(p_o)\,\frac{dV_f}{dt}\ ,$$

$$\Delta\dot{m} = \dot{m}_1 - \dot{m}_9\ , \tag{C.1}$$

$$\dot{m}_9 = f(p_o,\ K_7,\ N)\ ,$$

$$\frac{dQ}{dt} - \dot{m}_9\,\Delta h_{9-1} - \Delta\dot{m}\,h_1 = (\overline{mc}\,\frac{dT_o}{dp_o})\ \frac{dp_o}{dt} + F_1(p_o)\,\frac{dV_f}{dt} + F_2(p_o,V_f)\,\frac{dp_o}{dt}$$

These equations have been derived in Appendix B. Combining and rewriting gives

$$f_1\,\frac{dp_o}{dt} - f_2\,\frac{dV_f}{dt} = \dot{m}_1 - \dot{m}_9\ , \tag{C.2}$$

$$F_3\,\frac{dp_o}{dt} + F_1\,\frac{dV_f}{dt} = -\,h_1\,\dot{m}_1 - H\,\dot{m}_q + \frac{dQ}{dt}\ , \tag{C.3}$$

$$\text{and}\ \dot{m}_q = f(p_o,\ K_7,\ N)\ , \tag{C.4}$$

where $H = \Delta h_{9-1} - h_1$

and $F_3 = F_2 + \overline{mc}\,\frac{dT_o}{dp_o}$

A control system is required to maintain the boiler pressure p_o and liquid volume V_f near some desired values $\overline{p}_o$, $\overline{V}_f$. The means to achieve this control is the regulation of the rate of heat added, $\frac{dQ}{dt}$, and the feed pump bypass throttle value, K_7, as functions of p_o, V_f, engine flow rate $\dot{m}_1$, and, perhaps, engine speed N. These functions are referred to as "control laws"; it is the objective of this section to develop a basis for designing these control laws.

For convenience, the performance of the control system can be divided into a steady-state and a transient component. The obvious measure of steady-state behavior is simply the magnitude of the steady-state errors $(p_o - \bar{p}_o)$ and $(V_f - \bar{V}_f)$. (This implies that our control laws are of such a form that model (C.2, 3, 4) does in fact approach steady-state conditions; if this condition is not met, then the steady-state values referred to should be understood as approximations to an actual limit cycle condition.) A convenient measure of transient behavior is given by the roots of the characteristic equation resulting from linearizing the differential equations (C.2, 3, 4) about a particular steady-state operating condition.

To begin, let us restrict our attention to the following general structure of the control laws*:

$$\frac{dQ}{dt} = a_{11}(V_f - \bar{V}_f) + a_{12}(p_o - \bar{p}_o) + \beta\dot{m}_1 , \tag{C.5}$$

$$K_7 = a_{21}(V_f - \bar{V}_f) + a_{22}(p_o - \bar{p}_o) + g(\dot{m}_1, N), \tag{C.6}$$

where a_{ij} and β are constants and $g(\cdot)$ is a yet undetermined function. Our analysis of the complete system (C.2 - C.6) proceeds in two steps: determination of steady-state values of p_o, V_f;

---

*Another possibility would be to add "integral control", e.g., set

$$\frac{dQ}{dt} = a_{11}(V_f - \bar{V}_f) + a_{13}\int(V_f - \bar{V}_f)\,dt$$

and

$$K_7 = a_{22}(p_o - \bar{p}_o) + a_{23}\int(p_o - \bar{p}_o)\,dt$$

This would have the effect of reducing steady-state errors at the probable expense of the sacrifice of same quality of transient response. The analysis of the system with this type of control was not pursued further because of time limitations.

then calculation of characteristic roots from the linearization of the equations about these values. Subsequently, we consider $\dot{m}_1$ and N to be held constant. (Changes in $\dot{m}_1$ and N may be considered to be inputs or disturbances to the boiler control system.)

To determine steady-state conditions, set

$$\frac{dp_o}{dt} = \frac{dV_f}{dt} = 0$$

in Eqs. (C.2, 3), and arrive at the set of equations

$$\dot{m}_9 = f(p_o, K_7, N) = \dot{m}_1 \tag{C.7}$$

$$\Delta h_{9-1}\, \dot{m}_1 = \frac{dQ}{dt} \tag{C.8}$$

which are to be solved together with (C.5) and (C.6).

The resulting equations are:

$$a_{11}(\tilde{V}_f - \overline{V}_f) + a_{12}(\tilde{p}_o - \overline{p}_o) = (\Delta h_{9-1} - \beta)\, \dot{m}_1 \tag{C.9}$$

$$a_{21}(\tilde{V}_f - \overline{V}_f) + a_{22}(\tilde{p}_o - \overline{p}_o) = \hat{f}(p_o, \dot{m}_1, N) - g(m_1, N), \tag{C.10}$$

where the function $\hat{f}(\cdot)$ arises in the solution of Eq. (C.7) for K_7,

$$K_7 = \hat{f}(p_o, \dot{m}_1, N),$$

and $\tilde{V}_f$ and $\tilde{p}_o$ indicate steady-state values. Subsequently, we assume for purposes of steady-state analysis that K_7 is not an explicit function of p_o.

First, note that if we take

$$\beta = \Delta h_{9-1}, \tag{C.11}$$

$$\text{and} \quad g(\cdot) = f(\cdot) \tag{C.12}$$

then a zero steady-state error results:

$$\tilde{V}_f = \overline{V}_f\,,$$

$$\tilde{p}_o = \overline{p}_o\,,$$

provided only that $\det A \neq 0$

$$\text{where } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Thus the control law with β and g given by (C.11) and (C.12) is most desirable.

In the general case, the steady-state errors are:

$$\begin{bmatrix} \tilde{V}_f - \bar{V}_f \\ \tilde{p}_o - \bar{p}_f \end{bmatrix} = A^{-1} \begin{bmatrix} (\Delta h_{g-1} - \beta)\,\dot{m}_1 \\ \hat{f}(\dot{m}_1, N) - g(\dot{m}_1, N) \end{bmatrix} \quad (C.13)$$

For the particular control law with $a_{12} = a_{21} = 0$, the steady-state errors have the form

$$\tilde{V}_f - \bar{V}_f = \left(\frac{\Delta h_{g-1} - \beta}{a_{11}}\right) \dot{m}_1, \quad (C.14)$$

and

$$\tilde{p}_o - \bar{p}_o = \frac{1}{a_{22}} [\hat{f}(\dot{m}_1, N) - g(\dot{m}_1, N)] \quad (C.15)$$

the pressure error may be determined conveniently from a graph of the function $\hat{f}(\cdot)$. Figure (4.1) plots $\hat{f}(m_1, N)$ for three values of N. The value of pressure error $(\tilde{p}_o - \bar{p}_o)$ may be calculated by superimposing a plot of the control function $g(\cdot)$ as shown in the figure.

Note that there are two strategies for decreasing steady-state errors:

1. Approximating in the control law the conditions (C.11) and (C.12) as closely as possible.
2. Making the magnitudes of a_{11} and a_{22} as large as possible.

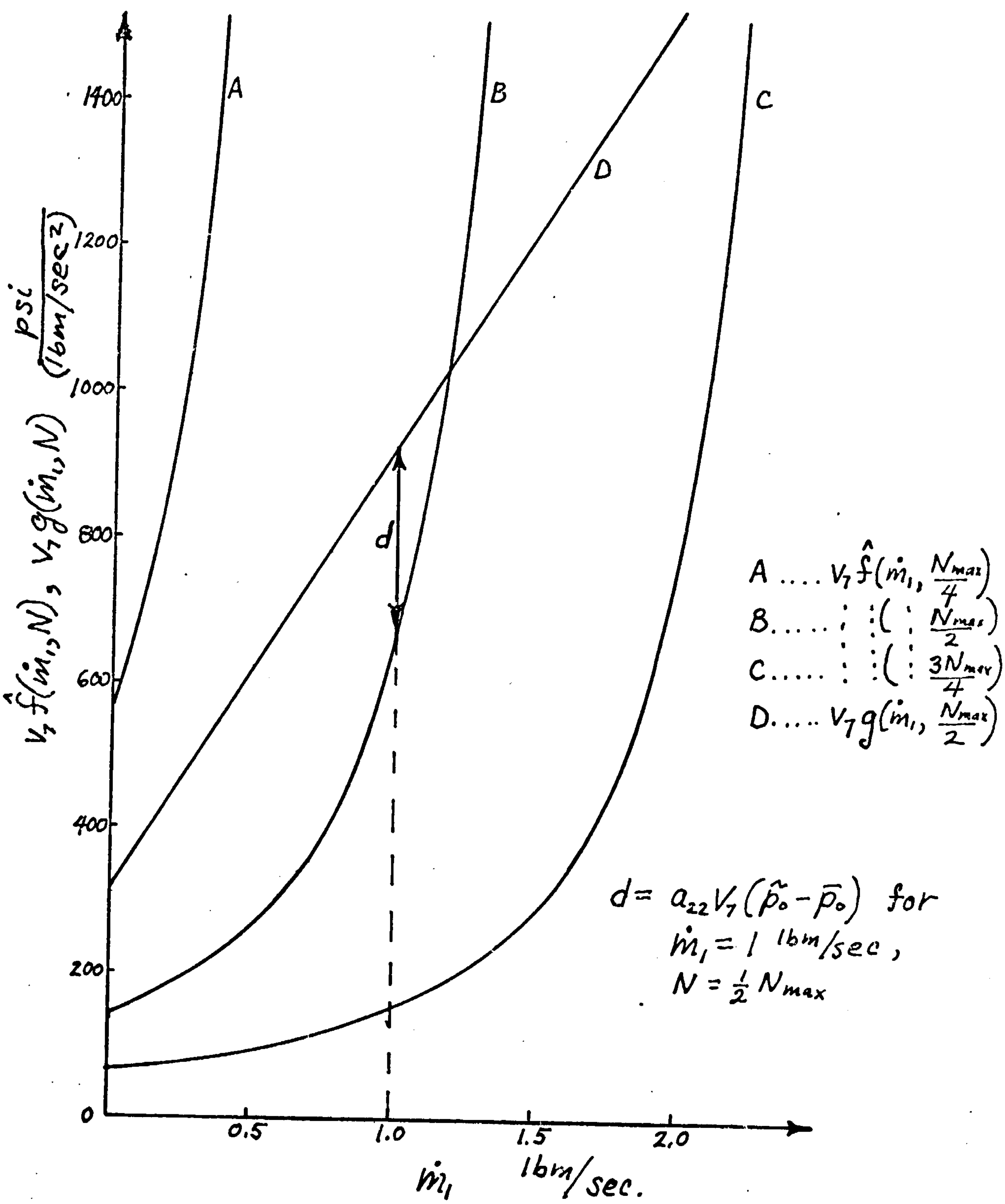


FIGURE 4.1
STEADY-STATE PRESSURE ERROR CALCULATION

Design considerations which arise here are difficulties in implementing $g(\dot{m}_1, N)$ and in producing large gains a_{11} and a_{22}; it is also possible that large values of a_{11} and a_{22} conflict with transient response objectives. In order to carry out a design one must consider, in addition to the allowable magnitude of the errors, the expected range of engine flow rates $\dot{m}_1$ and speed N.

Now,to treat the transient response, consider small deviations from steady-state values; define

$$
\left.\begin{aligned}
x_1 &\triangleq P_o - \tilde{P}_o, \\
x_2 &\triangleq V_f - \tilde{V}_f, \\
\delta\dot{m}_1 &\triangleq \dot{m}_1 - \tilde{\dot{m}}_1, \\
\delta k_7 &\triangleq k_7 - \tilde{k}_7 \\
\delta N &\triangleq N - \tilde{N}, \\
\delta q &\triangleq \frac{dQ}{dt} - \frac{d\tilde{Q}}{dt},
\end{aligned}\right. \qquad \text{(C.16)}
$$

where the ˜ refers to a set of (constant) steady-state reference conditions. Rewriting Eqs. (C.2, 3, 4) and (C.5, 6) in terms of the new perturbation variables (C.16) gives:

$$\tilde{F}_1 \frac{dx_1}{dt} - \tilde{F}_2 \frac{dx_2}{dt} = \tilde{\dot{m}}_1 + \delta\dot{m}_1 - \tilde{\dot{m}}_9 - \delta\dot{m}_9$$

$$\tilde{F}_3 \frac{dx_1}{dt} + \tilde{F}_1 \frac{dx_2}{dt} = - \tilde{h}_1(\tilde{\dot{m}}_1 + \delta\dot{m}_1) - \tilde{H}(\tilde{\dot{m}}_9 + \delta\dot{m}_9) + \frac{d\tilde{Q}}{dt} + \delta q$$

$$\delta\dot{m}_9 = \tilde{f}_{k_7}\, \delta k_7 + \tilde{f}_N\, \delta N + \tilde{f}_p\, x_1$$

$$\delta k_7 = a_{21}\, x_2 + a_{22}\, x_1 + \tilde{g}_{\dot{m}_1}\, \delta\dot{m}_1 + \tilde{g}_N\, \delta N$$

$$\delta q = a_{11}\, x_2 + a_{12}\, x_1 + \beta\, \delta\dot{m}_1$$

(Variables used as subscripts indicate partial differentiations) which become, in consideration of the steady-state Eqs. (4.7) and (4.8),

$$\tilde{f}_1 \frac{dx_1}{dt} + (\tilde{f}_p + \tilde{f}_{k7}\, a_{22})\, x_1 - \tilde{f}_2 \frac{dx_2}{dt} + \tilde{f}_{k7}\, a_{21}\, x_2 = (1 - \tilde{f}_{k7}\, \tilde{g}_{\dot{m}_1})\, \delta\dot{m}_1 - (\tilde{f}_N + \tilde{f}_{k7}\, \tilde{g}_N)\, \delta N \qquad (C.17)$$

$$\tilde{F}_3 \frac{dx_1}{dt} + (\tilde{H}f_p + \tilde{H}\tilde{f}_{k7}\, a_{22} - a_{12})\, x_1 + \tilde{F}_1 \frac{dx_2}{dt} + (\tilde{H}\tilde{f}_{k7}\, a_{21} - a_{11})\, x_2 = (\beta - \tilde{h}_1 - \tilde{H}\tilde{f}_{k7}\, \tilde{g}_{\dot{m}_1})\, \delta\dot{m}_1 - H(\tilde{f}_{k7}\, \tilde{g}_N + \tilde{f}_N)\, \delta N \qquad (C.18)$$

Taking Laplace transforms of Eqs. (4.17, 18) gives, in matrix form,

$$B(s)\, \underline{X}\, (s) = C\, \underline{Y} \qquad (C.19)$$

where

$$\underline{X} \triangleq \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

$$\underline{Y} \triangleq \begin{bmatrix} \Delta m_1(s) \\ \Delta N\, (s) \end{bmatrix}$$

$$B(s) \triangleq \begin{bmatrix} (\tilde{f}_1\, S + \tilde{f}_p + \tilde{f}_{k7}\, a_{22}) & (-\, \tilde{f}_2\, S + \tilde{f}_{k7}\, a_{21}) \\ (\tilde{F}_3\, S + \tilde{H}\, \tilde{f}_p + \tilde{H}\, \tilde{f}_{k7}\, a_{22} - a_{12}) & (\tilde{F}_1\, S + \tilde{H}\, \tilde{f}_{k7}\, a_{21} - a_{11}) \end{bmatrix}$$

and

$$C = \begin{bmatrix} (1 - \tilde{f}_{k7}\, \tilde{g}_{\dot{m}_1} & -\, (\tilde{f}_N + \tilde{f}_{k7}\, \tilde{g}_N) \\ (\beta - \tilde{h}_1 - \tilde{H}\, \tilde{f}_{k7}\, \tilde{g}_{\dot{m}_1}) & -\, H(\tilde{f}_N + \tilde{f}_{k7}\, \tilde{g}_N) \end{bmatrix}$$

The input-output relations can be expressed as:

$$\underline{X}(s) = T(s)\ \underline{Y}(s) \tag{C.20}$$

where T(s), the transfer function relating changes in engine flow rate and speed to boiler pressure and liquid volume, is:

$$T(s) = B(s)^{-1}\ C \tag{C.21}$$

The characteristic equation is:

$$\det\ [B(s)] = 0, \tag{C.22}$$

or

$$C_2 S^2 + C_1 S + C_o = 0,$$

$$C_2 = \tilde{f}_1\ \tilde{F}_1 + \tilde{f}_2\ \tilde{F}_3$$

$$C_1 = \tilde{f}_p\ \tilde{F}_1 + \tilde{f}_2\ \tilde{H}\ \tilde{f}_p - \tilde{f}_1\ a_{11} - \tilde{f}_2\ a_{12}$$
$$+ (\tilde{f}_1\ \tilde{H}\ \tilde{f}_{k7} - \tilde{F}_3\ \tilde{f}_{k7})\ a_{21}$$
$$+ (\tilde{f}_2\ \tilde{H}\ \tilde{f}_{k7} + \tilde{F}_1\ \tilde{f}_{k7})\ a_{22},$$

and

$$C_o = -\ \tilde{f}_p\ a_{11} - \tilde{f}_{k7}\ a_{22}\ a_{11} + \tilde{f}_{k7}\ a_{21}\ a_{12}$$

Note that some control is necessary to avoid a zero characteristic root, which occurs for $C_o = 0$. Also note that if only one control coefficient were allowed to be non-zero, then the only coefficient which could provide potentially satisfactory control (non-zero characteristic roots) is a_{11}. (Hence, the choice of particular control $a_{12} = a_{21} = 0$.)

The roots of (C.22) provide a means for describing the transient response, and thus Eq. (C.22) can be used as an aid in the selection of control-law coefficients. A realistic design should take into account the following factors:

1. Variation of coefficients with operating conditions. (Applies both to steady-state and transient analysis.)

2. Nonlinearities. The linear transient analysis based on Eq. (C.22) at best can provide only a first approximation to the actual system behavior, particularly if the control law must be implemented through strongly nonlinear devices, as it very likely would be. The nonlinear analysis would best be performed by computer simulation after specific hardware has been selected.